Exercise
Solve the problem below using Column Generation for linear programming

$$\begin{aligned} \text{maximize} \quad & 2x_1 + 4x_2 + x_3 \\ \text{subject to:} \quad & 2x_1 + x_2 + x_3 \leq 10 \\ & x_1 + x_2 - x_3 \leq 4 \\ & 0 \leq x_1 \leq 4,\ 0 \leq x_2 \leq 6,\ 1 \leq x_3 \leq 6 \end{aligned}$$

Master Problem

$$\text{maximize} \quad z = \sum_{j=1}^{p_R} (c^\top v_j)\lambda_j \tag{1}$$

$$\text{subject to:} \quad \sum_{j=1}^{p_R} (A_1 v_j)\lambda_j \leq 10 \tag{2}$$

$$\sum_{j=1}^{p_R} (A_2 v_j)\lambda_j \leq 4 \tag{3}$$

$$\sum_{j=1}^{p_R} \lambda_j = 1 \tag{4}$$

Consider μ_1, μ_2 e ν the dual variables related to the constraints 2, 3 and 4 respectively. p_R are the columns of the restricted master problem.
Auxiliary Problem

$$\begin{aligned} \text{maximize} \quad & cr = (2 - 2\mu_1 - \mu_2)x_1 + (4 - \mu_1 - \mu_2)x_2 + (1 - \mu_1 + \mu_2)x_3 - \nu \\ \text{subject to:} \quad & 0 \leq x_1 \leq 4,\ 0 \leq x_2 \leq 6,\ 1 \leq x_3 \leq 6 \end{aligned}$$

Let $x_1 = x_2 = 0,\ x_3 = 1$ be the initial solution. Master problem for column 1:

$$\begin{aligned} \text{maximize} \quad & z = 1\lambda_1 \\ \text{subject to:} \quad & 1\lambda_1 \leq 10 \\ & -1\lambda_1 \leq 4 \\ & \lambda_1 = 1 \end{aligned}$$